



Scale-Dependent Roughness and Multifractal Behavior of Topography in a Tectonically Active West Asian Domain

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Article Info	ABSTRACT
Article type: Research Article Article history: Received 2022/06/26 Received in revised 2022/11/20 Accepted 2022/12/15 Published 2022/12/16 Published online 2025/12/22 Keywords: Multifractal topography, Geomorphometry, Scaling analysis, Digital elevation models, Surface roughness, Morphotectonics.	Characterizing the scaling structure of topography is essential for linking observable land-surface morphology to the processes governing landscape evolution. Conventional geomorphometric approaches often describe terrain using self-affine models parameterized by a single roughness exponent, implicitly assuming statistical homogeneity across scales. Such formulations, however, may not adequately capture landscapes shaped by the coupled action of tectonic deformation, climatic forcing, and nonlinear erosional dynamics. In this study, we evaluate the scale dependence of surface roughness using high-resolution digital elevation data from the Iranian Plateau, a morphotectonically diverse region representative of actively evolving continental domains in West Asia. Fractal analysis based on box-counting yields a dimension of $D_f \approx 2.20$, while spectral analysis of elevation fields provides a Hurst-type roughness exponent of $\alpha \approx 0.48$. The lack of agreement with the canonical self-affine relation $D_f = 3 - \alpha$ indicates that the terrain cannot be characterized by a single scaling descriptor. To investigate this departure, we compute higher-order statistical measures of elevation variability, including generalized structure functions, multiscale height–height correlations, and scale-dependent curvature metrics. These analyses reveal that the scaling exponents vary systematically with statistical moment, demonstrating clear multiscaling and supporting a multifractal description of the surface. The results suggest that topographic organization in tectonically active settings reflects heterogeneous process interactions operating over a hierarchy of spatial scales, rather than uniform rough-surface behavior. This work underscores the limitations of single-exponent geomorphic parameterizations and highlights the value of multifractal diagnostics for quantitative terrain analysis, comparative morphotectonic studies, and improved representation of landscape complexity in Earth-surface models.
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Introduction

Geomorphology seeks to explain how the Earth's surface acquires its observable form through the interaction of tectonic forcing, climatic drivers, and surface processes acting across a hierarchy of spatial and temporal scales. Topography, in this context, is not merely a geometric description of relief but a measurable expression of dynamic Earth-surface organization. Quantitative characterization of this organization has increasingly relied on scaling concepts, particularly fractal geometry, which provides tools for describing irregular landforms whose structure varies with observational scale (Lovejoy & Schertzer, 1990; Lavallée et al., 1993). The fractal dimension has therefore been widely adopted as a compact descriptor linking geometric complexity to physical landscape properties. At present, a key practical question is whether such compact descriptors remain informative when landscapes are sampled by modern DEM products whose resolution, uncertainty structure, and processing workflows can vary substantially across space and scale. (Ince et al., 2024; Sun et al., 2024)

However, growing evidence suggests that many natural landscapes cannot be adequately described using a single scaling exponent. Instead, they exhibit multiscaling behavior in which statistical properties change systematically across scales, reflecting the coexistence of multiple geomorphic regimes. This has led to the adoption of multifractal frameworks capable of capturing heterogeneous spatial organization and intermittency in elevation fields (Gagnon et al., 2006; Jin et al., 2022). Recent studies emphasize that surface roughness itself may emerge from the competition between processes that generate relief and those that smooth it, implying that scaling metrics can encode process-level information rather than serving only as descriptive statistics (Doane, 2024). Operationally, multiscaling is diagnosed when scaling exponents depend on the statistical moment (or order) used to probe elevation increments—i.e., when higher-order statistics do not collapse onto a single exponent as expected for a purely self-affine surface. (Jin et al., 2022; Doane, 2024)

The rapid expansion of high-resolution global elevation datasets has renewed attention to how scaling properties are measured and interpreted. Newly developed terrain products, including merged global elevation models and derived morphometric datasets, now allow land-surface variability to be evaluated consistently across regions (Ince et al., 2024; Sun et al., 2024). Similarly, specialized terrain reconstructions such as coastal digital terrain models demonstrate how data integration across measurement platforms can reveal previously unresolved morphological complexity (Pronk et al., 2024). These advances raise an important methodological consideration: whether observed scaling relationships reflect intrinsic geomorphic structure or artifacts of data resolution, processing strategy, and uncertainty propagation. This consideration is particularly relevant for scaling analyses, because apparent power laws can be influenced by resampling, detrending, and noise floors, thereby complicating the interpretation of “universal” exponents unless uncertainty and scale limits are explicitly acknowledged. (Li et al., 2025; Memon et al., 2025)

Recent geomorphological investigations have therefore begun reassessing scaling analysis within a data-aware framework. Studies examining DEM uncertainty, comparison methodologies, and dataset evaluation highlight that terrain metrics are sensitive to vertical

accuracy, spatial resolution, and differencing approaches (Li et al., 2025; Memon et al., 2025). Parallel developments in global geomorphic mapping—including large-scale submarine landform compilations and planetary surface classifications—demonstrate that fractal-based metrics remain powerful when carefully integrated with process interpretation (Yu et al., 2025; Rabethge et al., 2025). These findings collectively suggest that revisiting classical fractal approaches under modern data conditions can yield new insights into landscape organization. In this context, it is increasingly important to couple “classic” first-order descriptors (e.g., fractal dimension or spectral roughness) with higher-order statistics that can distinguish genuine multiscaling from resolution-driven effects. (Trevisani, 2025; Li et al., 2025)

Within this evolving methodological landscape, examining regions characterized by strong tectonic gradients and physiographic diversity offers an opportunity to evaluate whether multifractal descriptions provide a more realistic representation of terrain structure than monofractal models. Landscapes shaped by active deformation, lithologic contrasts, and climatic variability are particularly suited to testing whether scale-dependent statistical behavior corresponds to geomorphic complexity rather than measurement noise. In such settings, the expectation is that elevation fields will display measurable departures from single-exponent scaling, reflecting the superposition of processes operating at different characteristic lengths. Accordingly, the analysis here is structured to (i) test internal consistency between commonly used self-affine relations linking fractal dimension and spectral roughness, and (ii) evaluate whether moment-dependent scaling emerges robustly across the accessible scale range. (Doane, 2024; Trevisani, 2025)

By applying fractal and multifractal analyses to digital elevation data of this region, the study investigates whether a single fractal dimension adequately characterizes the observed morphology or whether a spectrum of scaling exponents is required to capture its structure. The analysis is designed to interpret scaling behavior not as an abstract mathematical property but as a quantitative signature of interacting geomorphic processes acting across scales. Specifically, we combine box-counting estimates of fractal dimension with spectral roughness analysis, and we quantify multiscaling using generalized structure functions and multiscale height–height statistics; together these diagnostics provide a reproducible basis for evaluating when and why single-exponent terrain parameterizations break down. (Jin et al., 2022; Li et al., 2025) For geographic framing, we use “West Asia” in the standard regional sense adopted in international scholarship to denote the southwest Asian domain spanning the eastern Mediterranean–Anatolia to the Iranian Plateau and adjacent arid basins; this terminology is employed here to contextualize the study area tectonically and physiographically rather than to imply uniform coverage of the entire region by the analyzed dataset. (Rabethge et al., 2025; Yu et al., 2025)

The Iranian Plateau provides an instructive example of a tectonically active continental domain where pronounced relief gradients and diverse land-surface process regimes coexist. It lies within the Arabia–Eurasia collision system, a major intracontinental deformation zone accommodating convergence through distributed shortening, strike-slip faulting, and crustal thickening across Iran and adjacent regions (Allen et al., 2004; Vernant et al., 2004). In

geomorphological terms, this setting forms part of the broader geodynamic province commonly referred to as West Asia, a designation used here to denote a tectonically coherent southwest Asian domain rather than a political unit (Agard et al., 2011).

Iran extends approximately between 25° and 40° N latitude and 44° to 63.5° E longitude, encompassing a large portion of the Iranian Plateau and its surrounding mountain belts. The physiography is dominated by the Zagros and Alborz orogenic systems, internally drained basins, and extensive arid to semi-arid lowlands, producing strong elevation contrasts and marked spatial heterogeneity (Alavi, 2004; Mouthereau et al., 2012). These characteristics reflect the combined influence of active tectonics, lithologic contrasts, and climatic gradients, which together regulate erosion, sediment redistribution, and drainage organization across multiple spatial scales (Allen et al., 2013).

Elevation ranges from near sea level along marginal basins to peaks exceeding 5 km. Mount Damāvand, located in the Alborz Mountains, represents the highest summit in Iran at approximately 5.6 km above sea level and is associated with Quaternary volcanism linked to regional tectonic processes (Davidson et al., 2004). This wide vertical range contributes to substantial relief energy and enhances the detectability of scale-dependent geomorphic signatures in topographic datasets.

Taken together, the tectonic activity, physiographic diversity, and wide elevation range of the Iranian Plateau make it a suitable natural laboratory for examining whether observed terrain organization is adequately captured by a single scaling exponent or requires a multifractal description.

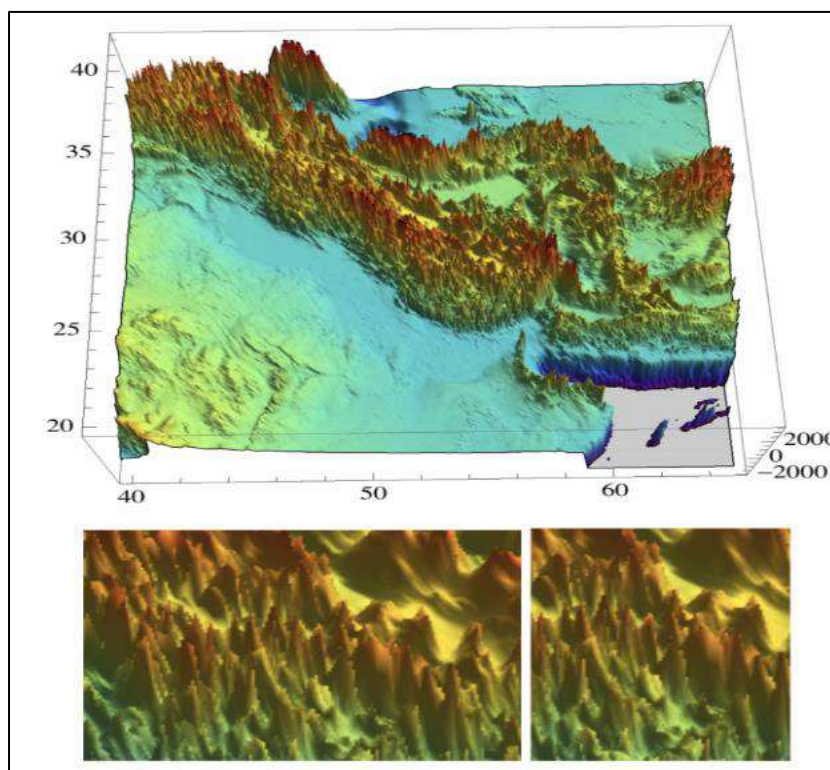


Figure 1: The topography of West Asia, which has been used for the fractal analysis in the present study (upper figure). The lower figures are two cuts from the upper image to show the scale invariance or fractal

property of the topography. It is impossible to guess the length scale of the images in these two cases without referring to the upper image for the topography with the given length scales.

Material and Methods

Fractal and multifractal analysis of topography has long been a very useful method to obtain synthetic topography in geology and geography which has led to a variety of different results. The concept of fractals was first introduced by Mandelbrot (1968) as a measure of the Earth's topography, the length of a rocky coast line (Mandelbrot, 1982). The scale invariance property of Earth's topography in a variety of geological terrains is the key notion of its fractality (See Fig. 1). Scale invariance means that the statistics of a portion of height profile in a window size of L is actually independent of the size L . This property leads to the emergence of power-laws like $Y \sim X^\alpha$ between two given quantities Y and X (where X is mostly referred to a measure of length-scale in the system) with the scaling exponent or fractal dimension α . The fractal dimension is a measure of global property of the system in question and in many cases is independent of various details and thus it is used to classify different systems and models in terms of the value of the fractal dimension besides other statistical measures.

However, in various studies in the past (See e.g., Lovejoy & Schertzer., 1990; Lavallee et al., 1993), it has been notified that in some cases the topography is not far than simple to be explained with a single scaling exponent such as the fractal dimension, it is rather more appropriate to study topography as a scale invariant quantity that generally requires multifractal measures and exponent functions. This gives rise to an infinity of fractal dimensions for different statistical moments of a variable which are needed to completely characterize the scaling properties. This means that for a height profile $h_i = \{h_1, h_2, h_3, \dots, h_n\}$ of a sample topography in a window size of L with N_L number of data points, if one measures the n th moment of the height fluctuation defined by:

$$m_n = \langle h^n \rangle = \frac{1}{N_L} \sum_{i=1}^{N_L} h_i^n \quad (1)$$

(in which the symbol $\langle \cdot \rangle$ denotes for the spatial averaging), with the observation that the second moment with $n = 2$ scales like $M_{2n} \sim L^{2\alpha}$, then for a mono-fractal topography, one expects that for the higher moments with $n=3,4,5,\dots$, the scaling exponent would be simply n times α , i.e.,

$$M_n \sim L^{n\alpha} \quad (2)$$

But for a multi-fractal height topography, one would observe that for each value of n the scaling relation produces a new fractal dimension or scaling exponent $\alpha(n)$ i.e.,

$$M_n \sim L^{\alpha(n)} \quad (3)$$

Where in general $\alpha(n) \neq n\alpha$.

A similar mono vs multifractal study exists for the artificial growth surface models (e.g., Morel et al., 2000). The growth models are studied in both the context of monofractality and multifractality (Bouchaud et al., 1993; Schmittbuhl & Vilotte, & Roux, 1995). Surface growth models and rough surfaces are interesting topics in geophysics and geography from both theoretical and experimental points of view (Barabasi & Stanley, 1997; Kondev &

Henley, & Salinas, 2000; Meakin, 1998). Various discrete models have been proposed to describe surface growth processes (see Barabási & Stanley, 1997; Kondev & Henley, & Salinas, 2000), which can be used to model Earth's topography. These models produce a self-affine height profile $h(x)$ such that its probability distribution function (PDF) remains invariant under scale transformation:

$$h(x) \cong b^{-\alpha} h(bx) \quad (4)$$

Where $\cong$ means statistically the same, and $\alpha \geq 0$. The exponent α is also called the roughness exponent. This relation means that if one looks at a window of size L in a topographical landscape and measures the height PDF, upon changing the window size by scale factor b , i.e., $(L \rightarrow bL)$, if the height profile is self-affine, then the PDF of the height profile in the new window times the scale factor $b^{-\alpha}$ should be the same as the original PDF. Such self-affine surface models with the single exponent α are called mono-fractal. For a self-affine topography, the fractal dimension of the height profile is known to be given:

$$\text{By } D_f = 3 - \alpha.$$

In this study, we apply various statistical methods to examine whether the Topography of the Iranian Plateau is scale-invariant mono-fractal or is a multifractal object with multiscale properties. To this end, we first measure the fractal dimension D_f of the Topography of the Iranian Plateau in section (2-1) by

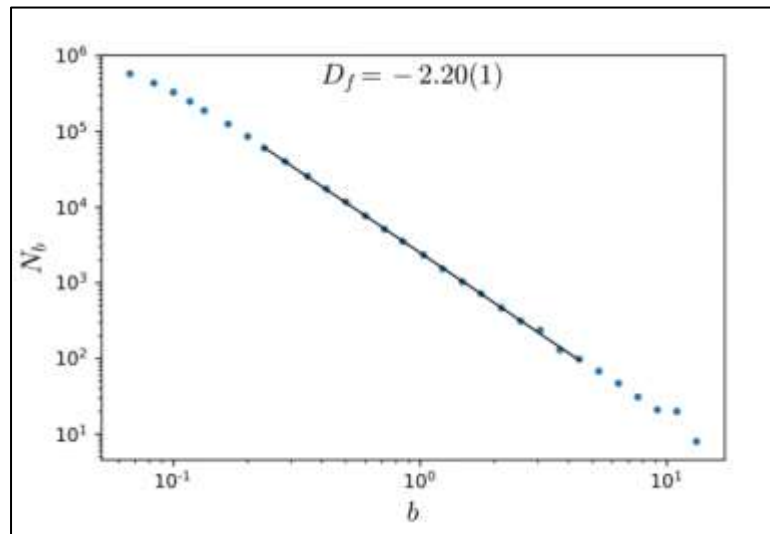


Figure. 2. The box-counting method was used to estimate the fractal dimension of the height topography in Iran. N_b stands for the number of boxes of size $b \times 1 \times 1$, to cover the part of the height columns. The solid line is the best power-law fit to our data (symbols), which gives the fractal dimension $D_f = 2.20(1)$. The number in the parentheses denotes the error bar for the last digit.

Using the box-counting method, next, in section (2-2), we obtain the roughness exponent of the Topography of the Iranian Plateau by the power spectrum analysis and check if it satisfies the relation $D_f = 3 - \alpha$ known for the self-affine surfaces. We find that this relation is violated, suggestive of the fact that the topography of the Iranian Plateau is actually multifractal and not a simple mono-fractal. In section (3-1), we also apply three different

methods to estimate the multi-fractal exponents $\alpha(n)$ introduced in Eq. 3 and confirm that $\alpha(n) \neq n\alpha$.

1.1. FRACTAL DIMENSION OF TOPOGRAPHY OF THE IRANIAN PLATEAU: BOX-COUNTING METHOD

In this section, we used the box-counting method to estimate the fractal dimension of the height profile in Iran. In this method, we considered the Topography of the Iranian Plateau shown in Fig. 1. We used the Topography of the Iranian Plateau grid data with high resolution on a square lattice of size 1320×1500 .

The heights $\{h_{i,j}\}$ are known at each grid node (i, j) distributed between the minimum height $\{h_{min}\} = -4398$ m and the maximum height $h_{max} = 5149$ m. In the box-counting method, we consider a cube of size $9547 \times 1320 \times 1500$ which covers all the height topography. The mesh sizes are scaled to unity. At each grid point (i, j) , we considered a height column of size $1 \times 1 \times h_{i,j}$ with $h_{min} \leq h_{i,j} \leq h_{max}$. We covered the entire cube with boxes of size $1 \times 1 \times b$, for different length scales b , and then count how many boxes of the grid, i.e., N_b , are covering part of the height columns. Then we do the same thing by using a finer grid with smaller boxes (smaller b). By shrinking the size of the grid repeatedly, we end up more accurately capturing the structure of the pattern of the topography. If the topography has a fractal structure, then one would expect the following power-law relation $N_b \sim b^{-D_f}$. (5)

Using the box counting method, the fractal dimension D_f of the height profile is thus given by the slope of the line when we plot the value of $\log(N_b)$ on the y-axis against the value of $\log(b)$ on the x axis i.e.,

$$D_f = -\frac{\log(N_b)}{\log(b)}. \quad (6)$$

As shown in Fig. 2, we find that the fractal dimension of the Topography of the Iranian Plateau is $D_f = 2.20(1)$. Within the present study, the numbers in the parenthesis denote the error bars for the last digits of the reported results.

1.2. THE ROUGHNESS EXPONENT OF TOPOGRAPHY OF THE IRANIAN PLATEAU: POWER-SPECTRUM ANALYSIS

The power spectral density (PSD) of a height profile is a useful mathematical tool that decomposes a topography into contributions from different spatial frequencies (wavevectors). From a mathematical point of view, the PSD is the Fourier transform of the autocorrelation function of the height profile, which contains just the power (and not the phase) across a range of wavevectors. This allows identification of the spatial frequencies that can be found in the topography and contains statistical information on the surface. To compute the power spectrum for the Topography of the Iranian Plateau, we first perform a discrete Fourier transform of the height profile $\{h_j\} = \{h_0, h_1, h_2, \dots, h_{N-1}\}$,

$$N j \tilde{h}_q = \sum_{j=0}^{N-1} h_j e^{-i \frac{2\pi q}{N} j} = \sum_{j=0}^{N-1} h_j \left[\cos\left(\frac{2\pi q}{N} j\right) - i \sin\left(\frac{2\pi q}{N} j\right) \right], \quad (7)$$

Where $i = \sqrt{-1}$. The power spectrum is then obtained by the average of the second moment of $|\tilde{h}_q|$

$$S_q = \langle |\tilde{h}_q|^2 \rangle. \quad (8)$$

For self-affine topographies with roughness exponent, the power spectrum exhibits a power-law dependence on the spatial frequency of roughness,

$$S_q \sim q^{m=-2(\alpha+1)}. \quad (9)$$

In Fig. 3, the power spectrum for the height profile of the Topography of the Iranian Plateau is plotted. As can be seen in the Figure, it shows a power-law relation with exponent $m = -2.96(1)$, which gives the roughness exponent $D_f = 0.48(1)$.

$$D_f = 3 - \alpha \quad (10)$$

The roughness exponent for the self-affine (mono fractal) surfaces is known to be related to the fractal dimension D_f of the height profile as

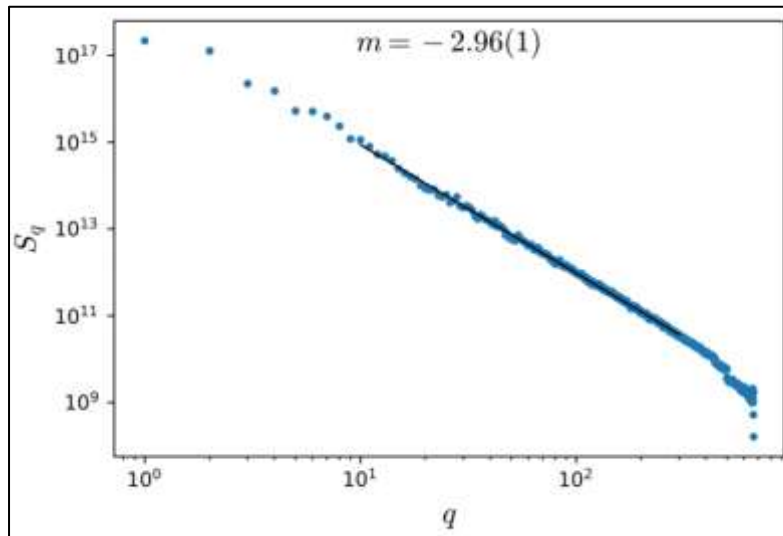


Figure 3. Power spectrum S_q of the height profile $\{h\}$ for Topography of the Iranian Plateau, versus q . For a self-affine surface, the slope of the curve in the log-log scales should be $M = -2(\alpha + 1)$ in terms of the roughness exponent α . We find that $M = -2.96(1)$ which yields $\alpha = 0.48(1)$.

If we examine this relation for the values $D_f = 2.20(1)$ and $\alpha = 0.48(1)$ obtained for Topography of the Iranian Plateau, it does not clearly hold. In the following section, we take this observation into account and apply various analyses to unravel the multifractal nature of the Topography of the Iranian Plateau.

Results and Discussion

1.1. MULTI-FRACTAL ANALYSIS OF TOPOGRAPHY OF THE IRANIAN PLATEAU

In this section, we examine three different analyses, including the total variance of the height profile in a box of size b as a function of b , i.e., $W_q(b)$ the generalized height-height correlation ¹(r D_q function²) b (C_q) and the scale-dependent curvature. We show the indications of multi-fractality in the statistics of the topography of the Iranian Plateau and compute the multi-fractal exponents $\alpha(n)$.

1.1.1. Total variance of the height profile, $W_q(b)$

The first quantity of interest is the generalized form of the height variance, defined as

$$W_q(b)W_q(b) = \langle |h(x) - \tilde{h}_b|^q \rangle b, \quad (11)$$

Where the symbol $\langle \cdot \rangle$ denotes the spatial averaging over the window box of size b , and $\tilde{h}_b$ stands for the average height in the window box of size b . For the self-affine topographies, this quantity has a power-law relation with the box size b ,

$$W_q(b) \sim b^{\alpha(q)} \quad (12)$$

In which the roughness exponent α is defined via the second moment with $q = 2$ as $W_2(b) \sim b^{2\alpha}$

$$\alpha(q) = q\alpha$$

The generalized exponent for the self-affine surfaces should behave linearly, i.e., $\alpha(q) = q\alpha$

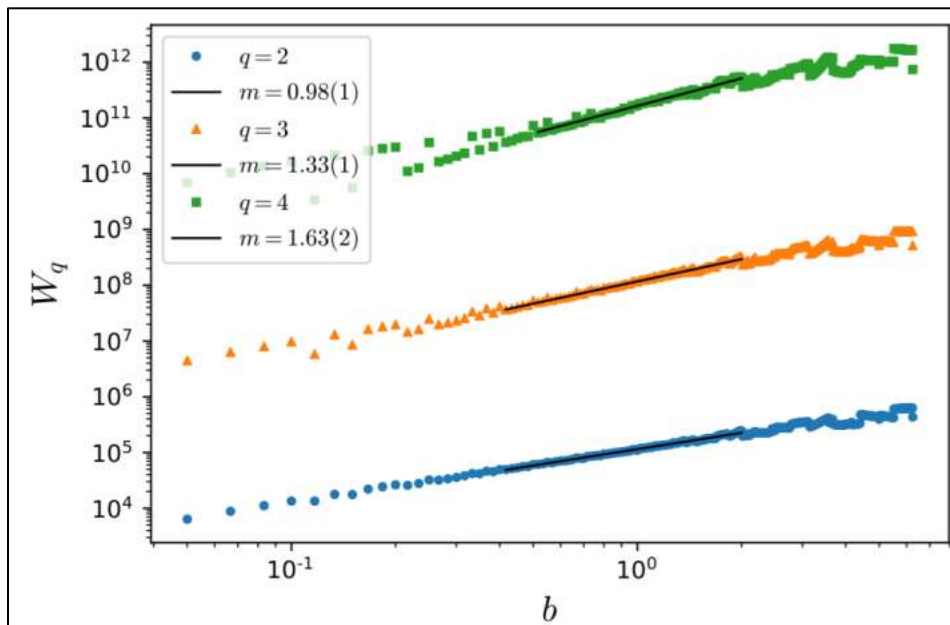


Figure 4. Total variance of the height profile, $W_q(b)$, measured in different boxes of size $q = 2; 3; 4$. before different moments $q = 2; 3; 4$. The corresponding exponents are obtained by measuring the slope of the log-log plots reported in the legend.

¹. Generalized height-height correlation

². Scale dependent curvature

The generalized exponent for the self-affine surfaces should behave linearly, i.e., $\alpha(q) = q \alpha$. For our data set for the height profile of Topography of the Iranian Plateau we consider boxes of different size $b \times b$ and first compute the average height within the box:

$\bar{h}_b = \frac{1}{b^2} \sum_{i,j=1}^b h(i,j)$. We then compute the scale-dependent variance $W_q(b)$ as a function of the length scale b explicitly by the following discrete sum,

$$W_q(b) = \frac{1}{b^2} \sum_{i,j=1}^b |h(i,j) - \bar{h}_b|^q \quad D_q(r) = \langle |h(x+r) - h(x)|^q \rangle \quad (13)$$

The results are reported in Fig. 4 for three different moments $q = 2; 3; 4$. We found a very good scaling relation when the results are plotted in a log-log scale. The best linear fit to our data, as shown by the solid lines in the Fig. 4, gives the generalized exponents $\alpha(2) = 0.98(1)$,

$\alpha(3) = 1.33(1)$ and $\alpha(4) = 1.63(2)$. One can simply read the roughness exponent from the second moment: $\alpha = (2)/2 = 0.49(1)$. If the topography of Iran is self-affine, one would expect that $\alpha(3)/3 = \alpha(4)/4 = \alpha$.

If we test this relation, we can find that $\alpha(3)/3 = 0.44(1)$ and $\alpha(4)/4 = 0.41(1)$, which is different from the roughness exponent obtained from the second moment: $\alpha = 0.49(1)$.

This result indicates that the statistical properties of the height topography of Iran is not self-affine and thus is not a simple fractal; it rather contains a rich multi-fractal structure.

1.1.2. Generalized height-height correlation function, $D_q(r)$.

The second quantity of interest to verify the multifractality of the Topography of the Iranian Plateau is the generalized height correlation function, $D_q(r)$, defined as,

$$D_q(r) = \langle |h(x+r) - h(x)|^q \rangle \quad (14)$$

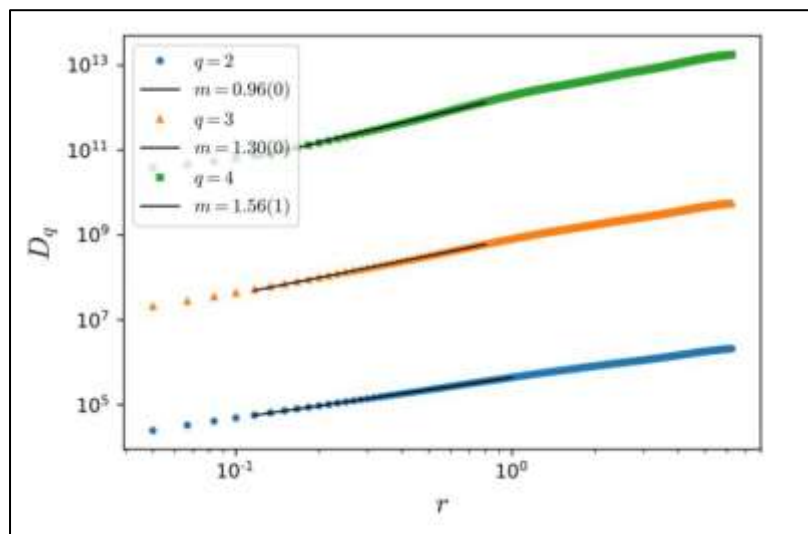


Figure 5. Generalized height-height correlation function, $D_q(r)$, for different moments $q = 2; 3; 4$. The corresponding exponents are obtained by measuring the slope of the log-log plots reported in the legend.

Table 1. The multifractal exponents $\alpha(q)$ measured for $q = 2, 3, 4$ by three different analysis i.e., Total variance of the height profile, $W_q(b)$ Generalized height-height correlation function, and scale-dependent curvature, $C_q(b)$

Method	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	$\alpha = \alpha(2)/2$
$W_q(b)$	0.98(1)	1.33(1)	1.63(2)	0.49(1)
$D_q(r)$	0.96(0)	1.30(0)	1.56(1)	0.48(0)
$C_q(b)$	0.99(0)	1.39(1)	1.53(1)	0.495(5)

We adopt the same definition on the lattice, including the whole data set to measure $D_q(r)$ and examine its scaling relation with r . For self-affine surfaces, one should observe the following power-law relation

$$D_q(r) \sim r^{\alpha(q)} \quad (15)$$

In which $\alpha(q) = q\alpha$, where $\alpha = \alpha(2)/2$. As we can see in Fig. 5, we find a very good scaling relation with r in the log-log scale. By linear fitting, we obtain the slopes of the linear fits, which are the generalized exponents for three different $q = 2, 3$, and 4 . We find $\alpha(2) = 0.96(0)$ giving the roughness exponent $\alpha = \alpha(2)/2 = 0.48(0)$. We also find the higher-order exponents $\alpha(3) = 1.30(0)$ and $\alpha(4) = 1.56(1)$. If we again check the linear relation to test the existence of self-affinity, we find that $(3)/3 = 0.43(1)$ and $(4)/4 = 0.39(1)$, which both deviate from the roughness exponent and are indeed indicative of a multifractal statistics of the height profile of the Topography of the Iranian Plateau.

1.1.3. Scale-dependent curvature, $C_q(b)$

To further check the efficacy of our previous computations and strengthen the evidence of multifractal properties in the topographical landscape of Iran, let us

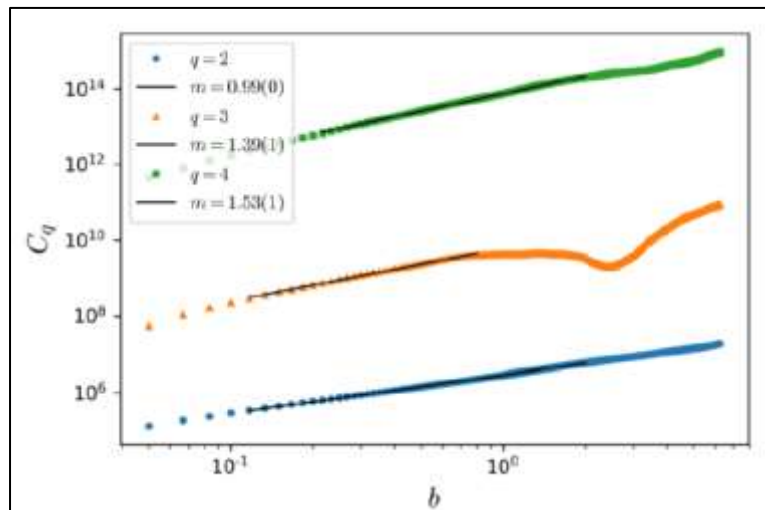


Figure 6. Scale-dependent curvature $C_q(b)$ measured in different boxes of size $b \times b$, for different moments $q = 2, 3, 4$. The corresponding exponents are obtained by measuring the slope of the log-log plots reported in the legend.

Now, we can compute the scale-dependent curvature, $C_q(b)$. The local curvature $C_b(x)$ at point x in a window of size b is defined as following

$$C_b(x) = \sum_{m=1}^M [h(x + be_m) - h(x)] \quad (16)$$

The offset directions $\{e_m\}_{m=1}^M$ are a fixed set of vectors summing to zero. In our case (square lattice), $\{e_m\}$ pointing along the $\{10\}$ type directions. To be more explicit, for each grid point $(i; j)$ on the square lattice, we define the local curvature $C(i; j)$ as

$$C(i,j) = [h(i+1, j) + h(i-1, j) - 2h(i, j)] + [h(i, j+1) + h(i, j-1) - 2h(i, j)] \quad (17)$$

We measure $C(i; j)$ for all lattice points $(i; j)$ belonging to a window size of $b \times b$. Then, for each moment q , we take a spatial average over the window box to obtain the scale-dependent curvature $C_q(b)$, i.e.,

$$C_q(b) = \langle C^q(i, j) \rangle_b = \frac{1}{b^2} \sum_{i,j=1}^b C^q(i, j). \quad (18)$$

For most topographical landscapes, the scale-dependent curvature satisfies the following power law relation

$$C_q(b) \sim b^{\alpha(q)} \quad (19)$$

In which, for $q = 2$, the roughness exponent α is obtained $\alpha(2) = 2\alpha$. For self-affine surfaces, the linear relation $\alpha(q) = q\alpha$ should hold different moments of the scale-dependent curvature of the Topography of the Iranian Plateau. The results are plotted in the log-log scale, which shows the power-law behavior in a wide range of length scales. The best fit to our data (shown by the solid lines) gives the multifractal exponents for each moment q . The exponents are listed in Table 1.

We find that $\alpha(2) = 0.99(0)$ which gives the roughness exponent $\alpha = \alpha(2)/2 = 0.495(5)$ in accord with our previous estimates obtained from different methods like the power spectrum analysis and two methods mentioned in subsections (3-1-A) and (3-1-B). We also find the higher moment exponents $\alpha(3) = 1.39(1)$ and $\alpha(4) = 1.53(1)$.

As mentioned above, for a self-affine surface, one should find $\alpha(q) = q\alpha$ or equivalently, $\alpha(q)/q = \alpha$. If we test this criterion for the Topography of the Iranian Plateau, we find that $\alpha(3)/3 = 0.46(1)$ and $\alpha(4)/4 = 0.38(1)$ which differ from our estimate for the roughness exponent $\alpha = 0.495(5)$. This completes our conclusion that the height profile of the Topography of the Iranian Plateau has multifractal statistics.

Conclusion

The application of monofractal and multifractal analysis is used for the correct analysis of random quantities describing drops and rises. In this study, the random quantity is altitude at any point in the topography of Iran. The meaning of drop and rise in altitude is that the drop and rise are proportional to the average altitude in the topography of Iran. Looking at the topographic profile, most altitudes are distributed around the average value. In fact, the average altitude is its first average torque. Therefore, it is assumed that the altitude profile is reduced by its average until the drop-and-rise profile of altitude is obtained. The second-rank torque, which is an indicator of deviation from the average, is represented by the

variance, by which the deviation from the criterion can be calculated; this criterion gives the amount of drop and rise around the average.

This drop and fall increase with the increasing scale of the studied topographic area. The reason is that on larger longitudinal scales, there are more elevation points available, and as a result, in this larger statistical community, the probability of finding a height that is far from the average is greater. Therefore, the logical expectation is that the criterion deviation has a direct relationship with the longitudinal scale of the study area. This suitability at the topographical level follows a power form; that is, the criterion deviation of altitude in an area with longitudinal scale L grows as L^α , where $0 \leq \alpha \leq 1$. The roughness of the surface is attributed to this exponent. The closer α is to zero, the rougher the surface is, with more drop and rise; and if α is closer to one, it indicates a smoother surface with much less drop and fall.

Therefore, if one considers lower-rank torques such as the average and variance of altitude in the drop–rise altitude profile, most information is obtained about altitudes close to the average altitude; conversely, higher-rank torques contain information related to very large drops and rises, far from the average. Therefore, if the difference in altitude from the average is $h-h^- = 0.1$ the torque of order 2 is proportional to $(h-h^-)^2 = 0.01$, while the 4th-rank torque, for example, is proportional to $(h-h^-)^4 = 0.0001$, which is negligible versus 0.01. Conversely, for large drops and rises with $h-h^- = 10$, the second-rank torque (the variance) is equal to 100, and the fourth-rank torque is equal to 10000, which is very significant compared with the second-rank torque. On the other hand, as mentioned, very large drops and rises are more likely at larger length scales, so higher-rank torques find particular importance at large length scales in geomorphological studies.

The main question is whether there is a relationship between the statistical analysis of a topographic landscape at a large scale (such as a country) and its analysis at a smaller longitudinal scale (like a topographic area). The response to this question depends on whether this rough surface is a self-similar monofractal surface or a multifractal surface. If this surface is monofractal, all higher-rank torques can be described based on the second-rank torque; therefore, large-scale studies do not need to analyze all the data and spend a great deal of time, energy, and cost. While, if it is a multifractal surface, it means that statistical analyses at regional scales are not easily generalizable to large scales. Thus, the importance of large drops and rises in altitude at these scales plays a determinative role. Therefore, the importance of the results obtained in this research is that it has been shown that the topographic altitude profile in Iran is not monofractal but is multifractal. Hence, the generalization of regional analysis is not valid from a specific range to a wider topographic range, and for more accurate analysis and more precise statistical information, all data over the wider area should be statistically analyzed within the explicitly defined scale range supported by the DEM and the analysis procedure.

Our findings help other research on Iran's topography in various areas of geology and geography to apply a regionally compatible approach for statistical analysis within a given length scale. This research is able to pave the path for further research on geometrical fractal

dimension in the fields of geography, geology, geotourism, environment, and other geosciences.

This study is consistent with the research works of (see, e.g., Lovejoy & Schertzer, 1990; Lavallee et al., 1993), (Bouchaud et al., 1993; Schmittbuhl, Vilotte, & Roux, 1995), (Barabasi & Stanley, 1997; Kondev, Henley, & Salinas, 2000; Meakin, 1998), and related multiscaling frameworks in topographic analysis (see Barabasi & Stanley, 1997; Kondev, Henley, & Salinas, 2000).

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